Adiabatic state preparation of interacting two-level systems

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We consider performing adiabatic rapid passage (ARP) using frequency-swept driving pulses to excite a collection of interacting two-level systems. Such a model arises in a wide range of many body quantum systems, such as cavity QED or quantum dots, where a nonlinear component couples to light. We analyze the one-dimensional case using the Jordan–Wigner transformation, as well as the mean field limit where the system is described by a Lipkin–Meshkov–Glick Hamiltonian. These limits provide complementary insights into the behavior of interacting systems under ARP, suggesting our results are generally applicable. We demonstrate that ARP can be used for state preparation in the presence of interactions, and identify the dependence of the required pulse shapes on the interaction strength. In general interactions increase the pulse bandwidth required for successful state transfer, introducing new restrictions on the pulse forms required.

Precise control of quantum mechanical systems is a sought after feature for applications in quantum information and investigations of quantum dynamics. Discrete atomic-like systems, or qubits, can be excited by using external field pulses that induce Rabi oscillations, with the final state determined by the intensity and duration of the pulse. However, this method is sensitive to fluctuations in the driving field, transition energy and other sources of disorder [1]. An alternative approach, which is robust against such variations, is the use of frequency-swept ("chirped") pulses to perform adiabatic rapid passage (ARP). In this method, the frequency of the driving field is swept through the transition to be excited, implementing the Landau–Zener process for adiabatic passage [2, 3]. Provided the gap induced by the applied field is large compared with the sweep rate the process is adiabatic, and the wavefunction is transferred from the initial ground state to the target state with high probability.

ARP is a well-established technique in nuclear magnetic resonance, where chirped radio frequency pulses are used to manipulate nuclear spins [4]. More recently, there have been a number of investigations into using ARP with optical pulses to control excitons in quantum dots [1, 5–7], including the creation of entangled states [8–10]. This has coincided with growing interest in producing many-body systems with strong light-matter interactions, such as coupled photon cavities or polaritonic systems [11]. A protocol such as ARP that allows robust control of the quantum state in these systems would enable the investigation of quantum dynamics in highly non-equilibrium regimes [12–15].

In established examples of ARP the interactions are weak on the scale of the level splittings generated by the ARP pulse, and hence the former can be straightforwardly neglected. The aim of this paper is to demonstrate how ARP may be extended to strongly-interacting regimes where this is not the case. We consider a model of interacting two-level systems which, by comparison to the case of a single two-level system [1], allows the effect of interactions to be identified. We show that ARP remains an effective approach in the interacting case, provided the pulse bandwidth is sufficient to span the spectrum of the collective modes generated by the interactions. Although our model is relatively simple, our results are relevant across a wide range of systems, including cavity QED systems [11], quantum dots [10, 16], superconducting qubits [17, 18] and doped impurities in semiconductors [19].

The model we consider consists of a set of N interacting two-level systems driven by an external field (in the rotating wave approximation):

$$H = \sum_i \left[\frac{E}{2}(\sigma_i^z + 1) + (f_i(t)\sigma_i^+ + \text{h.c.}) \right] - \sum_{i,j} J_{ij}\sigma_i^+\sigma_j^-, \tag{1}$$

where $\boldsymbol{\sigma}_i$ are the Pauli matrices for the two-level system i, and $\sigma_i^\pm = (\sigma_i^x \pm i\sigma_i^y)/2$. In this form the two states $\sigma_i^z = \pm 1$ are understood to correspond to the presence or absence of an excitation of the ith two-level system, e.g., of an exciton in a particular state of a particular quantum dot. We will also refer to the collective pseudospin $\boldsymbol{S} = \sum_i \boldsymbol{\sigma}_i/2$, whose z-component is related to the total excitation or occupation $n = S_z + N/2$. $f_i(t)$ is the coherent external pulse used to perform ARP, and J_{ij} is the interaction between systems i and j. At this stage we assume that the energy of the excitation $E > 0$ is the same for all transitions, and neglect interactions of the form $\sigma_i^z\sigma_j^z$. This model could be realized in precisely engineered cavity [11] or circuit QED [17, 18] systems. Furthermore, a less idealized model of this form can be used to describe many realizations of interacting qubits, such as coupled quantum dots [10, 16]. These systems often exhibit disorder in the energies E and interaction strengths J_{ij}, but the robustness of ARP means the gen-

eral understanding we obtain of the effect of interactions is applicable.

Decomposing the driving field into amplitude and frequency $f_i(t) = g_i(t)\exp(i\int\omega(t')dt')$, and eliminating the instantaneous frequency from the driving term using a unitary transformation, the Hamiltonian becomes:

$$H = \sum_i \left[\frac{(E-\omega(t))}{2}(\sigma_i^z+1) + (g_i(t)\sigma_i^+ + \text{h.c.})\right] - \sum_{i,j} J_{ij}\sigma_i^+\sigma_j^-. \quad (2)$$

For the discussion in this paper, we consider a Gaussian, linearly chirped pulse with uniform amplitude,

$$g_i(t) = g\exp(-t^2/\tau^2), \quad \omega(t) = E + \alpha t, \quad (3)$$

where g parametrizes the pulse amplitude, τ is the temporal width of the pulse and α is the linear chirp. We discuss the pulse and system parameters in terms of the dimensionless combinations $g\tau$, $J\tau$ and $\alpha\tau^2$ ($\hbar = 1$). For $\alpha = 0$, Eq. (3) becomes a Rabi pulse centered at frequency E, with a pulse area proportional to $g\tau$.

In the non-interacting case where $J = 0$, the use of ARP to transfer a two-level system from the ground state $\sigma^z = -1$ to the excited state is well understood [1]. For $g = 0$, the energies of the two levels cross when $E + \omega(t) = 0$. The presence of the field $g \neq 0$ produces an anticrossing and the adiabatic state smoothly varies from the initial ground state to the excited state. When the pulse amplitude is time independent, $g(t) = g$, the model reduces to the canonical Landau-Zener problem [2, 3]. The probability of remaining in the adiabatic state (and so being transferred from the initial ground state to the excited state) is $1 - \exp(-2\pi g^2/\alpha)$, so that the final population is always increased by reducing the chirp α, increasing the adiabaticity of the process. In the case of ARP, using pulses of finite duration, $g(t)$ is no longer constant. Thus, in order for adiabatic passage to occur, the two levels of the system must be coupled together long enough that the character of the eigenstates changes sufficiently slowly. This introduces the requirement that $\alpha \gg 1/\tau^2$ [1]. In the limit $\alpha \to 0$, the system undergoes Rabi oscillations rather than ARP.

In order to understand how this process generalizes to the interacting case, we first examine a one-dimensional chain with nearest neighbor interaction $J_{ij} = J\delta_{i,i+1}$. In this case, the energy levels for $g = 0$ can be determined using the Jordan-Wigner transformation $\sigma_i^z = 2c_i^\dagger c_i - 1$, $\sigma_i^- = \exp(i\pi\sum_{j<i} c_j^\dagger c_j)c_i = T_i c_i$ where c_i are fermionic operators [20]. After also performing a Fourier transformation the Hamiltonian, Eq. (2), becomes:

$$H = -\sum_k [\alpha t + J\cos k]c_k^\dagger c_k + \frac{1}{\sqrt{N}}\sum_{k,i}(g_i^* T_i c_k e^{ikr_i} + \text{h.c.}), \quad (4)$$

where N is the number of sites and $k = -\pi + 2\pi m/N$ with m integer. The Jordan-Wigner transformation has previously been used to describe Landau-Zener transitions for anisotropic spin chains in a changing magnetic field [21, 22]. The Landau-Zener transitions in that model result from the anisotropy, which affects the subspaces spanned by fermion operators of a given $|k|$ independently. In contrast, the spatial dependence of the nonlinear T_i term in our model leads to terms in the Hamiltonian that couple fermion states with different $|k|$.

In the Jordan-Wigner representation the different energy eigenstates correspond to different occupations of the fermion states. The completely empty (spin down) state corresponds to the vacuum with no fermions $|0\rangle$. Likewise, the completely occupied (spin up) state corresponds to the case with all fermion states filled, $\prod_k c_k^\dagger|0\rangle$. For large $|t|$ the first term in Eq. (4) dominates and the eigenstates, shown in Fig. 1 for $N = 4$, are split into $N + 1$ bands labelled by the total number of fermions, which physically corresponds to the excited-state population of the two-level systems n.

In the non-interacting case, the energy levels are independent of k and all the states in the nth band have energy $-n\alpha t$. The presence of interactions, $J \neq 0$, lifts the complete degeneracy of states within each band as shown in Fig. 1. The separate states correspond to the different allowed values of the total spin $\boldsymbol{S}^2$ for a given S^z. In order to prepare a fully occupied state, the quantum state must then be transferred via multiple level crossings from the $n = 0$ to $n = N$ bands [23, 24].

The splitting of the level crossings that allows adiabatic state transfer is introduced by the external pulse field g. In the general case there will be some variation in the driving field between the two-level systems, which makes the form of the coupling term in the fermionic representation complicated to determine due to the nonlocality of the Jordan-Wigner string T_i. However, for the uniform driving we consider $[g_i(t) = g(t)]$, the coupling to the field in the non-transformed Hamiltonian, Eq. (2), can be rewritten $g(t)\sum_i(\sigma_i^+ + \sigma_i^-) = g(t)(S^+ + S^-)$, and the transitions therefore conserve $\boldsymbol{S}^2$. If the system starts in the ground state, it will thus always be in an eigenstate of $\boldsymbol{S}^2$ with the maximal eigenvalue $S(S+1)$ where $S = N/2$. There is one state in each band with this value of $\boldsymbol{S}^2$, and the transitions between these states have matrix elements $g\sqrt{(N-n)\left[(n+1/2)\pm 1/2\right]}$. The corresponding states in each band are the most symmetrical states, which for $J > 0$ ($J < 0$) have the lowest (highest) energies, see Fig. 1 (Fig. 1 inset) [25].

The field term in the Hamiltonian only changes the number of fermions, n, by ± 1. An anticrossing between non-adjacent bands can be induced by higher order virtual transitions. For example, a $n \to (n+2)$th band transition is possible via an intermediate $(n+1)$th band state. These higher-order interactions are suppressed in the mean-field limit discussed below [24, 26], but do play

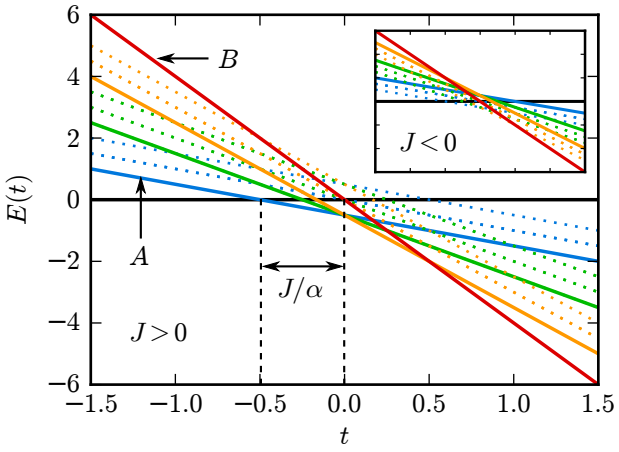

FIG. 1. (Color online) Time-dependent eigenenergies of the one-dimensional chain, Eq. (4), with $g = 0$, $J = \frac{1}{2}$ and $N = 4$ sites. Different colors indicate bands corresponding to different values for the occupation n of the two-level systems. Bold lines show the states in each band with maximum $\boldsymbol{S}^2$, which are coupled together when the pump is spatially uniform. Dashed lines are other eigenstates of the system, which have different values of $\boldsymbol{S}^2$. The vertical dotted lines show the separation in time between the crossing of the $n = 0$ band with the $n = 1$ (A) and $n = N$ (B) states. Inset: As main figure but with $J = -\frac{1}{2}$. Note that the order of crossings of the coupled (bold) levels has reversed.

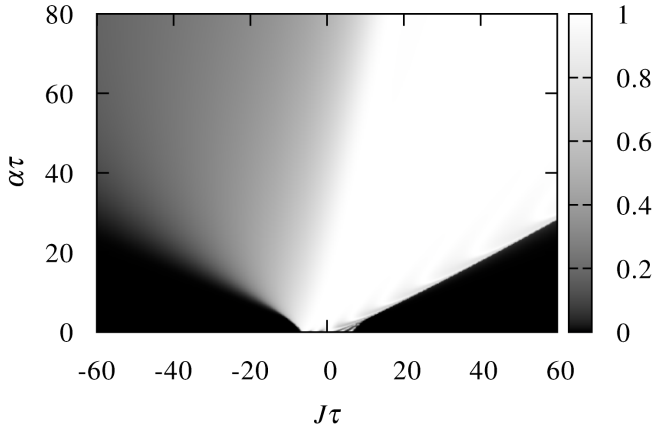

FIG. 2. Mean-field calculation of the average excitation of a set of interacting two-level systems, Eq. (2), driven from its ground state by the chirped pulse, Eq. (3) with $g\tau = 3$. $J\tau$ is the dimensionless interaction strength, and $\alpha\tau$ the dimensionless chirp. The regions where no excitations are created are a result of the finite duration of the pulse τ: as the chirp α is reduced, the level crossings of Fig. 1 no longer occur within the pulse and so adiabatic transfer is not possible. The boundary of this region is approximately $\alpha \propto J$.

a role in ARP if the connectivity is small.

The Jordan-Wigner transformation is only usefully applicable for the special case of nearest-neighbor hopping in one dimension. In higher dimensions, an alternative approach is to use the mean field approximation, which is exact in the limit $N \to \infty$, $J_{ij} = J/N^2$. We show in Fig. 2 results for the final occupation obtained using a spatially uniform pulse, Eq. (3), calculated by solving the Heisenberg equations of motion using the mean field replacement $\sum_{ij} J_{ij}\sigma_i^+\sigma_j^- = \sum_i J_{\rm eff}(\sigma_i^+ \langle \sigma_i^- \rangle + h.c.)$. In this approximation the Hamiltonian, Eq. (2), can be rewritten in terms of the collective spin operators as the Lipkin-Meshkov-Glick Hamiltonian:

$$\begin{aligned} H_{MF} &= -J_{\rm eff}(S^+S^- + S^-S^+) - \frac{\alpha t}{2}S^z + 2gS^x \\ &= 2J_{\rm eff}(S^z)^2 - \frac{\alpha t}{2}S^z + 2g(t)S^x, \end{aligned} \tag{5}$$

where we have used $S^2 = (S^z)^2 + (S^+S^- + S^-S^+)/2$ and dropped terms which do not affect the dynamics. This Hamiltonian, with a time independent $g(t) = g$, has been used to describe Landau-Zener tunneling for a bosonic Josephson junction [24, 26–30].

Figure 2 consists of a fan of non-zero occupation bounded by large regions of essentially zero occupation. Within the fan, as in the normal Landau-Zener problem, increasing α decreases the final occupation as the increased velocity of the level crossing reduces the adiabaticity of the transition.

For a fixed value of α, the occupation within the fan increases (decreases) for $J > 0$ ($J < 0$). This variation corresponds to the changing relative positions of the level crossings, visible in Fig. 1 for the one-dimensional chain. In the absence of interactions, $J = 0$, the states within each band are degenerate and so all levels cross simultaneously at $t = 0$. As the interaction strength J is increased, the level crossings separate in time. As each level degeneracy becomes more isolated, the size of the anticrossing caused by g increases. When $J > 0$ and $\alpha > 0$ the crossings occur in "ascending order", i.e. the $n \to n + 1$ crossing occurs before the $n + 1 \to n + 2$ crossing. The increase in splitting due to the isolation of crossings then improves the efficiency of transfer to the occupied state, producing the increase of the occupation shown in Fig. 2. If $J < 0$, however, then for positive chirp the crossings occur in the "wrong order" (see Fig. 1, inset), so that it becomes more difficult for the system state to transfer via a series of transitions through adjacent bands, suppressing the probability of full occupation. In the mean field limit, this ordering leads to the formation of a swallowtail in the energy level evolution which causes a breakdown of adiabaticity and a corresponding reduction in the occupation [24, 27–29].

The large regions of zero occupation that define the fan are a result of the time dependence of the optical pulse $g(t)$ used to perform ARP. In order for the state to be transferred at a level crossing, the anticrossings caused by the field $g(t)$ must be large enough compared to the level velocity α to make the process adiabatic. Because

ARP uses pulses of a finite duration there is only a limited window during which $g(t)$ meets this criterion. With no interaction, $J = 0$, all crossings occur simultaneously and so may all occur within the window. In the presence of interactions, eventually the time of the first level crossing $J/\alpha \sim \tau$, and it will be pushed out of the pulse. Neglecting the change in anticrossing size discussed above and the effect of higher order, virtual transitions, the crossover to non-adiabatic behavior occurs along a line $J \propto \alpha$, which is approximately what is seen in Fig. 2. Below this line, the behavior is no longer adiabatic and the system undergoes more complicated dynamics, reducing to non-linear Rabi oscillations for $\alpha = 0$. Thus a pulse used to perform ARP in a large system must have a sufficient duration τ that it includes the entire region of level crossings separated by the interaction.

In conclusion, we have shown the consequences of intersystem interaction on using ARP to fully occupy a set of two level systems. The interaction lifts the degeneracy of the eigenstates of the Hamiltonian, Eq. (1), and causes the level crossings at which state transfer occurs to separate in time. As in the bosonic Josephson junction [24, 27–29], the isolation of each degeneracy increases the effective splitting so that adiabatic transfer can be achieved for larger chirps than in the non-interacting system. However, the separation of level crossings also introduces the additional condition for ARP that the pulse duration (or chirp) should be large enough to include all the necessary crossings, increasing with J. Physically, this occurs because the interactions broaden the spectrum into a set of collective modes forming a path from the ground to final states, and the pulse must cover this spectrum for the state preparation to be effective.

Although in this paper we have focused on the ideal case of uniform E, J and coupling g_i, our results apply more generally. Fluctuations in E and J will change the energies and character of the intermediate states so that they are not delocalized across the system. However, for $|t| \to \infty$, the highest and lowest states remain the empty or full states, so our results will still apply. Variation in g_i changes the size of splittings at a level crossing but, with the exception of fine-tuned cases, anticrossings will still form, allowing adiabatic transfer.

As the model discussed in this paper represents limits of more complicated many-body systems including the Bose-Hubbard, Dicke or Jaynes-Cummings-Hubbard models [11, 17, 31], these results can be used as a basis for understanding the behavior of ARP in these models. It could then be used as a robust method of preparing far-from-equilibrium states in those systems for use in quantum information contexts or as equivalents of the quantum quenches performed in ultracold atomic gases.

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